Lens Rotation Preliminary Analysis

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Datasets and procedure

To analyze the effect of the rotation of the lens in the image formation two datasets were acquired.

- **Calibration Dataset** - composed by 9 calibration images (of a chessboard grid). The captured images were obtained with the calibration box and the back light. The camera was fixed and the lens was rotated to several angles. This dataset aims to study the movement of the principal point with the variations in the lens rotation.
- **White Images Dataset** - composed by 17 images totally white. The captured images were obtained shooting the back light. The camera was fixed and the lens was rotated to several angles. This dataset aims to study the movement of the conic center and lens mark angle with the variation in the lens rotation.

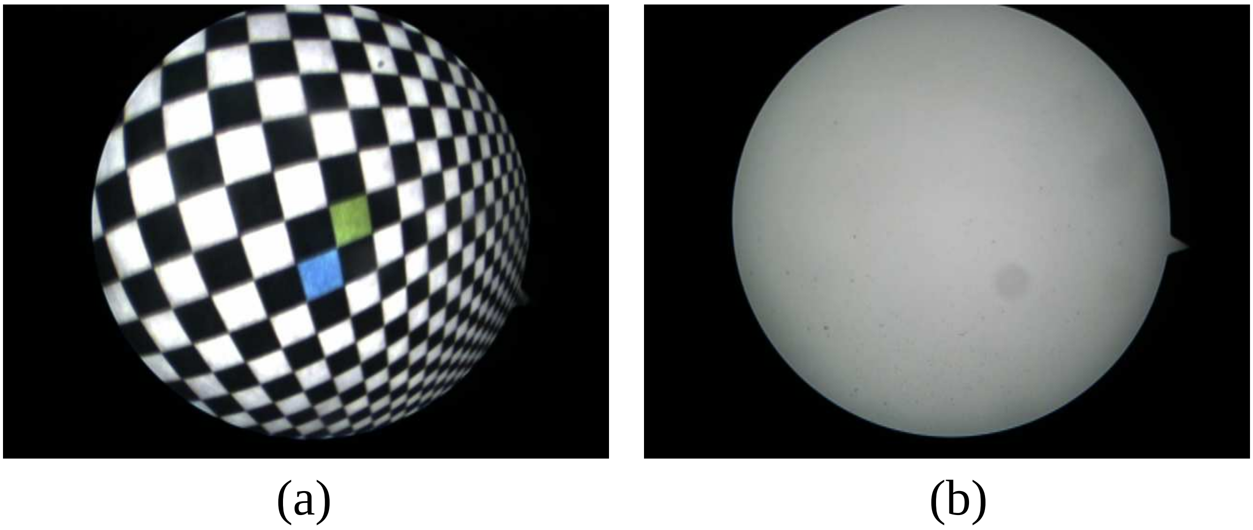
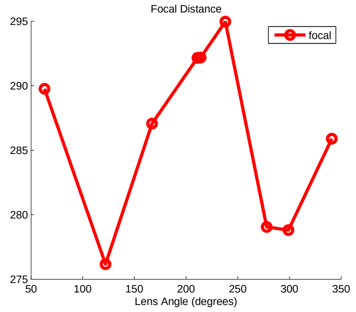


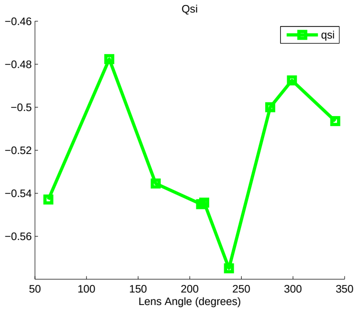
Figure 1: Datasets example images. (a) - calibration dataset image; (b) - white dataset image.

For the *Calibration Dataset*, all images generated a different calibration. The calibration was refined using the first-order non-linear optimization image by image. For the **White Images Dataset**, the boundary and the lens mark were detected.

Lens rotation behavior results

We observed that the lens mark, the estimated boundary conic center and the principal point of the images move according to a circular trajectory around the same point with the rotation of the lens probe. In this case the rotation center is located at $(317, 407)$ approximately.





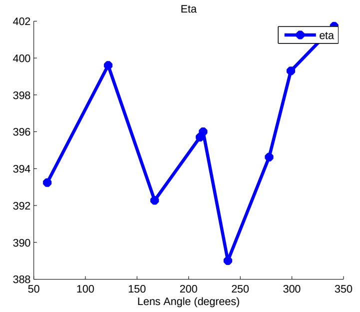
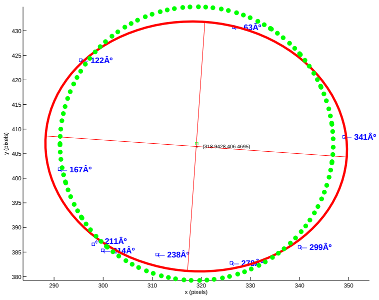


Figure 2: Focal distance variation with the lens rotation.

Figure 3: Distortion parameter variation with the lens rotation.

Figure 4: $f/\sqrt{-\xi}$ variation with the lens rotation.



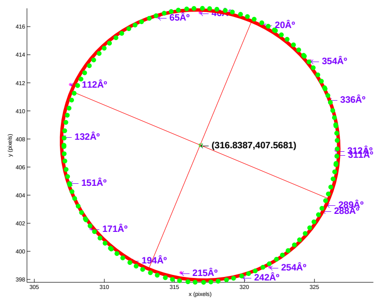
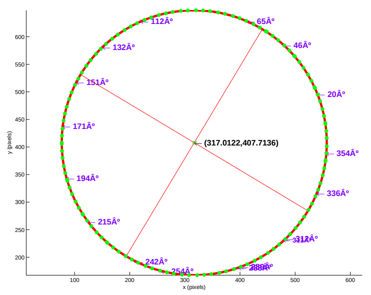


Figure 5: Principal point variation with the lens rotation.

Figure 6: Estimated conic center variation with the lens rotation.



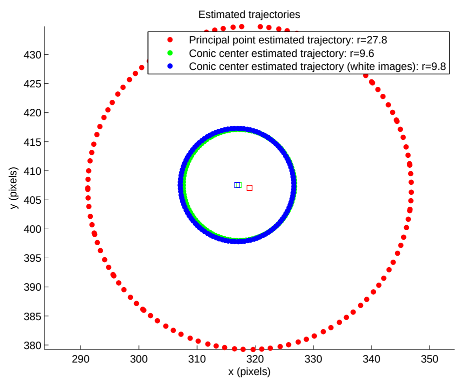


Figure 7: Lens mark position variation with the lens rotation.

Figure 8: Estimated trajectories (considering the circle fitting)

Radial distortion correction with rotation compensation

We will consider that there is a reference image for which the camera is fully calibrated. We know:

$$K_0 = \begin{bmatrix} f & 0 & c_x \\ 0 & f & c_y \\ 0 & 0 & 1 \end{bmatrix} \qquad \Omega_0 = \text{the surrounding circle} \qquad P_0 = \text{the image coordinates of the lens mark} \quad (1)$$

Note that for K_0 we are considering a=1 and s=0. We can think in the projection as the lower camera (CCD) projecting an image acquired by the upper camera (probe). In this case the projection matrix can be decoupled in the following way:

$$X_i \sim \begin{bmatrix} fc & 0 & c_x \\ 0 & fc & c_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} fb & 0 & 0 \\ 0 & fb & 0 \\ 0 & 0 & 1 \end{bmatrix} f_\xi(X_c) \tag{2}$$

The CCD camera acquires the borescope image that we can think as a "virtual" image in a front-parallel plane. Concerning the reference image we have

$$K_0 = \begin{bmatrix} f & 0 & c_x \\ 0 & f & c_y \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} fc & 0 & c_x \\ 0 & fc & c_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} fb & 0 & 0 \\ 0 & fb & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3}$$

Which means that $f = f_c f_b$. We will assume that the relative rotation between the boreoscope and the CCD camera is around an axis orthogonal to the plane passing by $q\prime$. We can model the problem as follows:

$$X_i \sim \begin{bmatrix} fc & 0 & cx \\ 0 & fc & cy \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\theta) & \sin(\theta) & (1-\cos(\theta))q\prime_x - \sin(\theta)q\prime_y \\ -\sin(\theta) & \cos(\theta) & \sin(\theta)q\prime_x + (1-\cos(\theta))q\prime_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} fb & 0 & 0 \\ 0 & fb & 0 \\ 0 & 0 & 1 \end{bmatrix} f_\xi(\cdots) \tag{4}$$

We can rewrite the equation as:

$$X_i \sim \begin{bmatrix} \cos(\theta) & \sin(\theta) & (1-\cos(\theta))q_x - \sin(\theta)q_y \\ -\sin(\theta) & \cos(\theta) & \sin(\theta)q_x + (1-\cos(\theta))q_y \\ 0 & 0 & 1 \end{bmatrix} K_0 f_\xi(\cdots) \qquad q_x = f_c \times q\prime_x + c_x \qquad q_y = f_c \times q\prime_y + c_y \tag{5}$$

We conclude that the rotation of the probe causes a rotation of the image around a point Q. The model for the radial distortion correction of any image with lens rotation θ is then presented as:

$$X_d = R_{(-\theta,q)} K_\eta f_{\xi=-1}\left(R_{(\theta,q)} K_f^{-1} X_p\right) \tag{6}$$

Real time correction strategy

- We will assume that the boundary is well approximated by a circle.
- We know K_0, Ω_0 and P_0 (initial intrinsic matrix, initial boundary and initial lens mark image coordinates) from an initial calibration image.
- We will track the curve Ω_i and, whenever possible, the point Pi.
- Ω_i and P_i are measures fed to an Extended Kalman Filter to estimate the rotation θ_i as well as the rotation center Q_i.
- From Ω_i we will get the center (O_x^i, O_y^i) and radius r_i of the current boundary.
- From the calibration image we get P_0, (O_x^0, O_y^0) and r_0 of the initial boundary.
- Note that from the centers only (of the initial boundary and the current boundary) it is impossible to compute Q and θ. We have to use the image coordinates of the lens mark to compute the rotation angle and center.

The EKF can then be formulated as follow:

Model Equation:

$$\begin{bmatrix} \theta \\ \dot{\theta} \\ q_x \\ q_y \\ r \end{bmatrix}_k = \begin{bmatrix} 1 & h & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \\ q_x \\ q_y \\ r \end{bmatrix}_{k-1} + Q_k \tag{7}$$

Measure Equation:

$$\begin{bmatrix} O_x \\ O_y \\ P_x \\ P_y \\ r \end{bmatrix}_k \begin{bmatrix} \cos(\theta)O_x^0 + \sin(\theta)O_y^0 + (1-\cos(\theta))q_x - \sin(\theta)q_y \\ -\sin(\theta)O_x^0 + \cos(\theta)O_y^0 + \sin(\theta)q_x + (1-\cos(\theta))q_y \\ \cos(\theta)P_x^0 + \sin(\theta)P_y^0 + (1-\cos(\theta))q_x - \sin(\theta)q_y \\ -\sin(\theta)P_x^0 + \cos(\theta)P_y^0 + \sin(\theta)q_x + (1-\cos(\theta))q_y \\ r_0 \end{bmatrix} + R_k \tag{8}$$

Practical results

We concluded that the rotation compensation of the lens probe greatly improve the radial distortion correction.

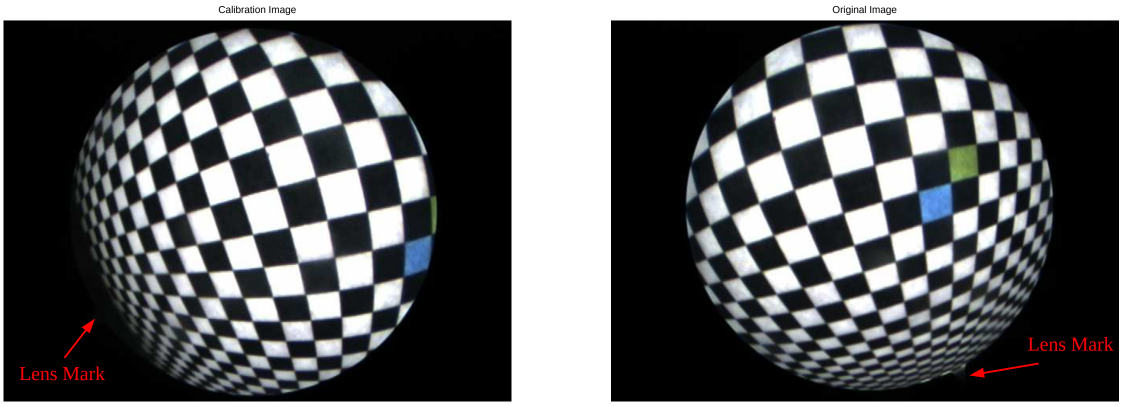


Figure 9: Reference image used to calibrate the camera (left) and test image which will be corrected with and without rotation compensation. Note that the two image have different probe rotation.

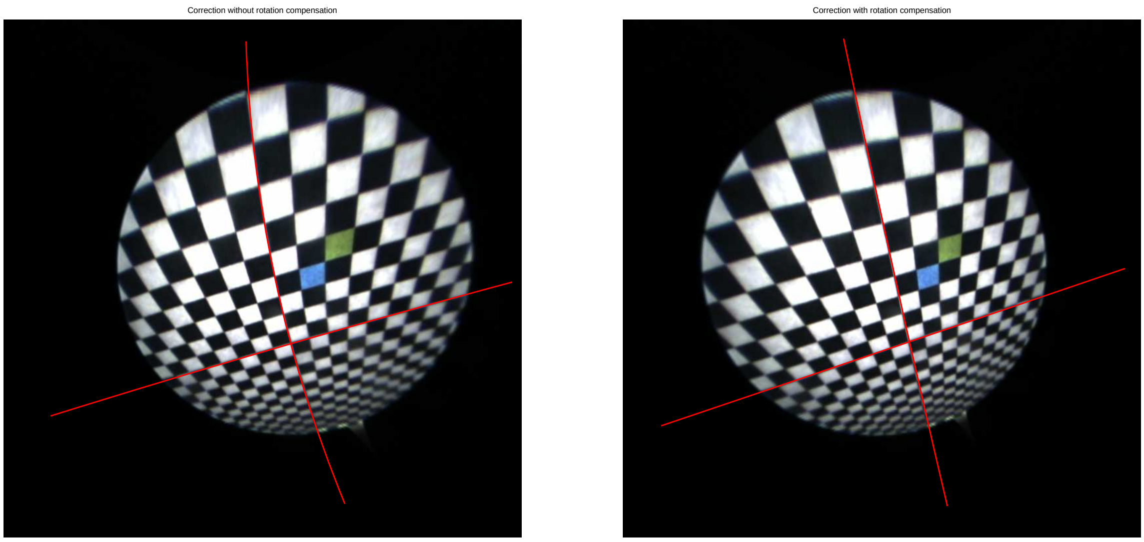


Figure 10: Corrected image without rotation compensation (left) and with rotation compensation (right).